Boundary Element Characterization of Coplanar Waveguides

Taek-Kyung Lee, Hao Ling, *Member, IEEE,* and Tatsuo Itoh, *Fellow, IEEE*

Abstract—**The quasi-static capacitance and inductance of the coplanar waveguide (CPW) are characterized independently using the boundary element method (BEM). The inductance calculation utilizes the magnetic scalar potential and avoids the usual vector formulation. The proposed method can be easily extended to characterize three-dimensional problems such as CPW discontinuities.**

I. Introduction

IN microwave and millimeter wave integrated circuits, coplanar waveguide (CPW) is widely used because it offers convenient ground connection, small crosstalk, and small radiation loss at discontinuities [1]. Several methods to obtain the quasi-static characteristics of the CPW have been developed by using only the capacitance calculation [2], [3]. However, to characterize CPW discontinuities by equivalent circuit parameters, it becomes necessary to calculate the inductance as well as the capacitance associated with the discontinuity [4]–[6].

In this letter, we present a boundary element formulation for independently calculating the quasi-static capacitance and inductance of a uniform CPW transmission line. The capacitance problem is first formulated using a boundary integral equation of the electric scalar potential. In an analogous manner, the inductance problem is formulated by utilizing the magnetic scalar potential. The proposed method has the advantage that inductance can be calculated through a simple scalar formulation, while other existing methods require the calculation of the magnetic vector potential. This feature should be most attractive when this method is extended to attack three-dimensional CPW discontinuity problems.

II. Capacitance of the Coplanar Waveguide

Fig. 1 shows a symmetric coplanar waveguide to be analyzed in which the substrate is composed of the nonmagnetic material. The thickness of the conductors is assumed zero and the relative dielectric constant of the substrate is denoted by ϵ_r. For the calculation of per-unit-length capacitance, the potential at the center strip is assumed to be 1 V and those at the ground conductors, 0 V. In each region, the electric scalar potential ϕ satisfies Laplace's equation, and can be expressed as a boundary integral equation from Green's identity. Moving the observation point r onto the boundary Γ, one can write the boundary integral equation for the scalar potential ϕ in each region as [7]

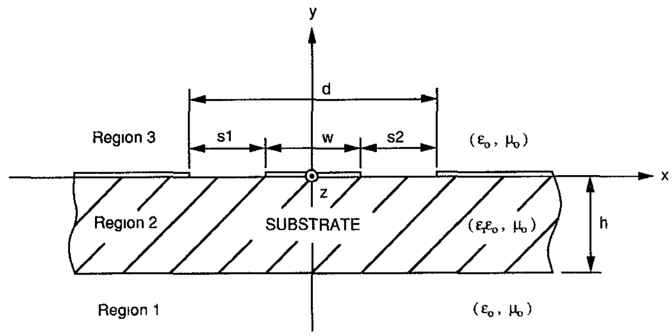

Fig. 1. Symmetric coplanar waveguide and its BEM regions.

$$\frac{1}{2}\phi(r) = \int_\Gamma \left\{ G(r,r')\frac{\partial \phi(r')}{\partial n} - \phi(r')\frac{\partial G(r,r')}{\partial n} \right\} d\Gamma, \quad r, r' \in \Gamma, \quad (1)$$

$$G(r,r') = -\frac{1}{2\pi}\ln|r'-r|,$$

$$\frac{\partial G(r,r')}{\partial n} = -\frac{\hat{n}\cdot(r'-r)}{2\pi|r'-r|^2}.$$

Here, $\hat{n}$ is the outward unit normal vector. At each air-substrate interface, the boundary conditions are

$$[\phi]_{\text{air}} = [\phi]_{\text{substrate}} \text{ and } \left[\frac{\partial \phi}{\partial n}\right]_{\text{air}} = -\epsilon_r\left[\frac{\partial \phi}{\partial n}\right]_{\text{substrate}}. \quad (2)$$

Applying the standard boundary element technique to the integral equation (1) and matching the boundary conditions, we can solve for the normal derivative of the potential on the boundaries.

Since the charge density on the ground conductor should be negligible far away from the edge of the ground conductor, the integration in (1) can be truncated at $x = D$, where $D \gg d$. Fig. 2 shows the characteristic impedance changes with respect to the normalized center conductor width (w/d) for $\epsilon_r = 9.8$, s1 = s2, and $h/d = 0.15$. The linear basis function and the collocation scheme are employed. The results of the boundary element method are compared with

Manuscript received June 7, 1991. This work was supported in part by the National Science Foundation Grant ECS-8657524. Editorial decision made by S. Maas.

T.-K. Lee and H. Ling are with the Department of Electrical and Computer Engineering, University of Texas at Austin, Austin, TX 78712.
T. Itoh is with the Department of Electrical Engineering, University of California, Los Angeles, 66-147A Engineering IV, 405 Hilgard Avenue, Los Angeles, CA 90024-1594.

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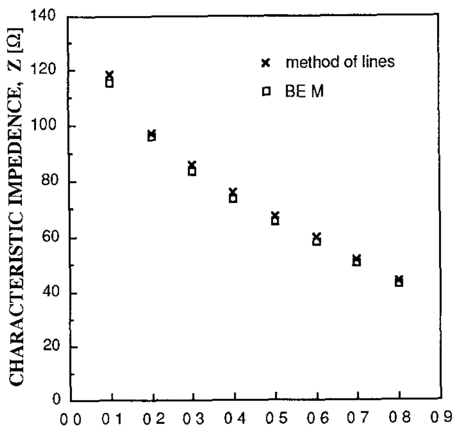

Fig. 2. Characteristic impedance Z of the coplanar waveguide for $s1 = s2$ and $h/d = 0.15$.

those of the method of the lines [8] and the errors are less than 2.7% when the number of the node is 184.

III. Inductance of the Coplanar Waveguide

In the capacitance calculation of the coplanar waveguide, the charge density on the center strip is obtained from the boundary integral equation of the scalar potential and its normal derivative. In a similar manner, we will now treat the inductance problem via a boundary integral formulation of the magnetic scalar potential. The magnetic scalar potential ψ is defined as

$$\psi(r) = -\int_{\text{ref.}}^{r} H \cdot dl, \tag{3}$$

where the zero potential reference can be chosen arbitrarily. Since ψ satisfies Laplace's equation, the following boundary integral equation can be obtained from Green's identity:

$$\frac{1}{2}\psi(r) = \int_{\Gamma} \left\{ G(r, r')\frac{\partial \psi(r')}{\partial n} - \psi(r')\frac{\partial G(r, r')}{\partial n} \right\} d\Gamma, \quad r, r' \in \Gamma. \tag{4}$$

To determine the proper excitation condition for the inductance calculation, we will assume that a current I flows in the $-z$-direction along the center strip of the coplanar waveguide and a current $I/2$ flows in the $+z$-direction on each of the ground conductors. We also will assume that no magnetic flux penetrates the conductor strips for the quasi-TEM mode and that $\partial\psi/\partial n$ vanishes on the strips. Since the magnetic flux lines encircle the center strip (in a clockwise fashion), the magnetic scalar potential defined in (3) is a multivalued function. We shall introduce a branch cut in the left gap, and assign a fixed potential of $I/4$ just above the gap, and a potential of $-3I/4$ just below the gap, as shown in Fig. 3. This ensures that the magnetic scalar potential jumps by I every time the cut is crossed in the clockwise direction. Next, it can be argued that the value of ψ in the right gap must be $-I/4$ from symmetry considerations. This is true for a coplanar waveguide with a nonmagnetic substrate, since the magnetic field is symmetrical about the $y = 0$ plane. In addition, as the magnetic flux lines are perpendicular to the gap, the magnetic scalar potential must be constant in the gap.

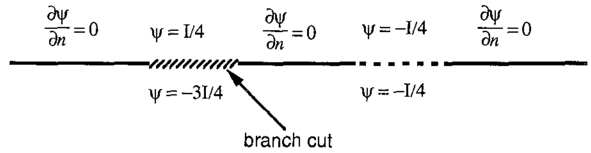

Fig. 3. Boundary conditions for the inductance calculation of the coplanar waveguide.

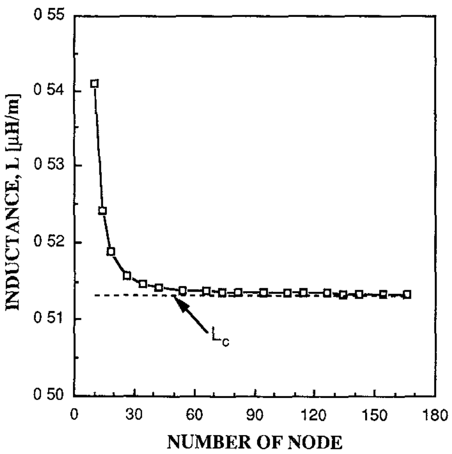

Fig. 4. Convergence of the inductance of the coplanar waveguide vs. number of node, where $s1 = s2$ and $w/d = 0.3$.

Since the normal derivative of the Green's function includes the scalar product $\hat{n} \cdot (r' - r)$ (see (1)), the second term of the integral equation (4) vanishes completely along the planar boundary. In addition, since $\partial\psi/\partial n$ vanishes on the conductor, the boundary integral equation (4) reduces to

$$\frac{1}{2}\psi(r) = \int_{\text{gap}} G(r, r')\frac{\partial \psi(r')}{\partial n} d\Gamma. \tag{5}$$

Note that the only unknown function in the above equation is $\partial\psi/\partial n$, or the normal component of the magnetic field, in the gaps. Again the boundary element technique can be used to numerically solve for $\partial\psi/\partial n$. Contrary to the capacitance calculation, it is only necessary to assign nodes in the gap region. Once the normal magnetic field in the gap is known, the magnetic flux density passing through the gaps is given by

$$B \cdot \hat{n} = \mu_o \frac{\partial \psi}{\partial n}, \tag{6}$$

where μ_o is the permeability of the free space. Finally, the inductance L is obtained from the ratio of the total magnetic flux φ penetrating the gap and the magnitude of the current

$$L = \frac{\varphi}{I}[H]. \tag{7}$$

Fig. 4 shows the convergence of the inductance calculation. As a check of the present formulation, we compare this result against that calculated indirectly from the capacitance of the line. The latter is given by the formula

$$L_c = \frac{1}{v_o^2 C_o}[H], \tag{8}$$

where C_o is the capacitance of the coplanar waveguide with $\epsilon_r = 1$ and v_o is the wave velocity in the free space. In Fig. 4, these two values agree very well. The difference is less than 0.1% for 154 nodes.

IV. Conclusion

A simple method for analyzing the capacitance and the inductance of the CPW are proposed and calculated by the boundary element method. The method uses the boundary integral equation of the electric and magnetic scalar potentials, and should be quite advantageous in the quasi-static characterization of CPW discontinuities. Furthermore, the present formulation can be easily extended to treat conductors with finite thickness.

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